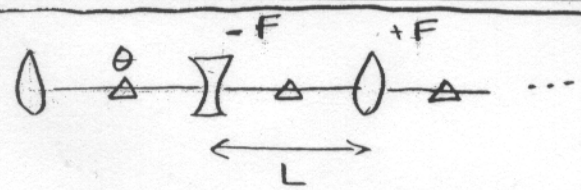
FODO Lattices – Natural chromaticity (MJS)



Phase advance per cell: μ ; $\sin \mu/2 = L/2F$

$$\hat{\beta} = 2F\sqrt{\frac{1+\sin\mu/2}{1-\sin\mu/2}} \qquad \hat{D} = \frac{L\theta}{\sin^2\mu/2}\left(1+\tfrac{1}{2}\sin\mu/2\right)$$

$$\check{\beta} = 2F\sqrt{\frac{1-\sin\mu/2}{1+\sin\mu/2}} \qquad \check{D} = \frac{L\theta}{\sin^2\mu/2}\left(1-\tfrac{1}{2}\sin\mu/2\right)$$

γ_t ...

$$\frac{1}{\gamma_t^2} = \left\langle \frac{D}{\rho} \right\rangle \approx \frac{L\theta}{\sin^2\mu/2}\cdot\frac{1}{\rho} \approx \frac{\theta^2}{\sin^2\mu/2}$$

$$\Rightarrow \gamma_t \simeq \frac{\sin\mu/2}{\theta} \approx \frac{\mu}{2\theta} \qquad \theta = \frac{2\pi}{N_{\frac{1}{2}\text{cells}}} = \frac{2\pi}{2\cdot N_{\text{cells}}}$$

$$\approx \frac{\mu}{2\pi}\cdot N_{\text{cells}}$$

$$\approx \nu$$

ξ_{nat} ...

$$\xi_{nat} = -\frac{1}{4\pi}\oint K\beta\, ds = -\frac{1}{4\pi}\sum \frac{1}{F_i}\beta_i$$

$$= -\frac{1}{4\pi} N_{\text{cells}}\left(\frac{\hat{\beta}}{F} - \frac{\check{\beta}}{F}\right)$$

$$= -\frac{N_{\text{cells}}}{4\pi}\left[2\sqrt{\frac{1+s}{1-s}} - 2\sqrt{\frac{1-s}{1+s}}\right]$$

$$= -\frac{N_{\text{cells}}}{2\pi}\left[\frac{1+s-(1-s)}{\sqrt{1-s^2}}\right] = -\frac{N_{\text{cells}}}{\pi}\,\frac{\sin\mu/2}{\cos\mu/2}$$

$$\approx -\frac{N_{\text{cells}}}{\pi}\cdot\frac{\mu}{2} \approx -\frac{\mu_{TOT}}{2\pi} = -\nu$$